

Surface oscillations in channeled snow flows.

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Abstract

An experimental device has been built to measure velocity profiles and friction laws in channeled snow flows. The measurements show that the velocity depends linearly on the vertical position in the flow and that the friction coefficient is a first-order polynomial in velocity (u) and thickness (h) of the flow. In all flows, oscillations on the surface of the flow were observed throughout the channel and measured at the location of the probes. The experimental results are confronted with a shallow water approach. Using a Saint-Venant modeling, we show that the flow is effectively uniform in the streamwise direction at the measurement location. We show that the surface oscillations produced by the Archimedes's screw at the top of the channel persist throughout the whole length of the channel and are the source of the measured oscillations. This last result provides good validation of the description of such channeled snow flows by a Saint-Venant modeling.

keywords: snow, channeled flows, surface oscillations, saint-venant equations

1 Introduction

In the past, many models have been used to describe the behavior of dense snow avalanches (Voellmy, 1955; Perla et al., 1980; Dent and Lang, 1983; Norem et al., 1987; Savage and Hutter, 1989; Naaim et al., 2003). Since snow avalanches are very destructive and cannot be controlled, it is very difficult to fully test these models experimentally. Direct measurements on avalanches do exist (Dent et al., 1998) but are rare and must be completed by "laboratory experiments" to study the rheology of snow (Nishimura and Maeno, 1989; Casassa et al., 1991; Kern et al., 2004). The main problem is that snow grains are very fragile. This brittleness makes it very difficult to use a large amount of snow in the laboratory. In consequence, as described in previous papers (Bouchet, 2003a; Bouchet et al., 2003b) we chose to work with an *in situ* rheometer, to use snow directly from the field. To stay as close as possible to naturally occurring flows in avalanches, we studied free surface snow flows in an inclined channel.

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Experiments over a wide range of slopes and thicknesses were performed to measure velocity profiles and friction laws in the flow. Results are fully reported in (Bouchet et al., 2004) and are briefly presented in section 2.2. One of the main purpose of the present article is to test the obtained snow flows with regards to a commonly used Saint-Venant model to see if the model can help in understanding and predicting the behavior of such flows.

As described later in the article the snow flow consists of a short first phase (2 to 3 seconds) of transient regime followed by a fully developed regime which lasts until the end of the experiment (around 30 seconds). The fully developed part of the flow is made on the one hand of a steady component non uniform in the streamwise direction but which tends to become uniform asymptotically (x-uniform asymptotic regime) as will be shown in the article. On the other hand, added to the steady component is a periodic one which will be studied in detail when looking to the surface oscillations that are present on top of the flow. The whole experimental and numerical results presented in the present paper are all focused on the fully developed regime leaving apart the transient one. The steady component of the fully developed flow is studied in detail in this article using the analytical and numerical results. As mentioned before, in all cases, oscillations on the surface of the flow were observed. The question of the origin of these oscillations has not been addressed up to now and is another main concern of this paper. Are they roll waves intrinsic to the flow like those observed in granular flows by Louge and Keast (2001)? Are they engendered by the feeding system as direct observations suggest? Numerical simulations and analytic solutions are used to get a better understanding of the flow characteristics.

Section 2 briefly presents the experimental set-up and results. The simulation technique is presented in section 3; it is based on the resolution of the shallow water Saint-Venant equations. The behavior of the flow with increasing streamwise distance, and in particular the way it approaches the x-uniform, fully developed regime, is then studied numerically and analytically. An estimate of the distance needed to attain the asymptotic regime is derived. Section 4 concerns the study of the oscillations on the surface of the flow, and the validation of the model.

2 Experimental set-up and results

The experimental facility and results are summarized in Bouchet et al. (2003b, 2004). A more detailed technical description of the set-up, and complete results, can be found in Bouchet (2003a).

2.1 The experimental device

A photograph of the experimental apparatus is shown in figure 1. The device consists of a 10 m long, 20 cm wide, 20 cm high channel, fed with snow by an Archimedes's screw from a hopper at the top of the channel. The screw is driven by a thermo-hydraulic engine, so that its rotation speed f is constant during an experimental run. Such a system is able to produce fully developed flows with a typical duration of 30 seconds. The floor of the channel is covered with sandpaper to create a rough surface. The slope angle of the channel, θ , ranges from 27° to 45° , in steps of 2° . A second adjustable parameter is the mean flow thickness (h) which is controlled by the value of the rotation speed f (f ranges from 0 rpm to 60 rpm). Twelve different flows were studied. Many types of snow can be found in the field. As a first study the repeatability of the experiments was privileged and thus flows were performed in the following conditions. In each case:

- to avoid snow melting the temperature of both snow and air was below -15°C ;
- snow grains were small and *rounded* with diameters between 0.2 and 0.4 mm;
- snow was sieved with a 3 cm mesh.



Figure 1: The experimental set-up is designed to perform free surface snow flows in a channel (length: 10 m, width: 20 cm, thickness: 20 cm). The coordinate system associated to describe the flows (see figure 4) is the following: the x axis is along the main direction of the channel, orientated downward; the z axis is perpendicular to the bottom of the channel, orientated towards the surface of the flow.

The velocity profile was measured 6 m downstream from the top, on one of the sidewalls. The flow thickness was measured at the same location (h_L) and also 1.5 m upstream ($h_{L-1.5}$).

Velocities are deduced (Dent et al., 1998) from the delay between signals at two identical opto-electronic sensors, one 2 cm downstream from the other. Each of these sensors consists of a photo-transistor and an LED. The latter emits an IR light beam which is reflected by the snow and collected by the photo-transistor. The signal produced by the photo-transistor is thus characteristic of the snow pack flowing in front of the sensor. Because the distance d between the two sensors is small enough to avoid deformation of the snow pack, the signals are identical but delayed by an amount of time τ . A cross-correlation method enables measurement of τ , and hence to deduce the velocity $v = d/\tau$.

The flow thickness measuring device (Leuze ODSM/V-5010-600-421) consists of an optical distance sensor positioned above the flow. It emits an IR light beam (a few degrees) which is reflected by the free surface of the flow and collected by a CCD sensor. The measured position of the reflected beam on the CCD sensor, is a function of the distance between the sensor and the surface. Knowing the distance between the sensor and the bottom of the flow allows the calculation of the flow thickness.

The main characteristics (slope angle, average thickness and frequency of the thickness oscillations) of the flows are listed in table 1. The frequency F of the oscillations was obtained by

slope angle (degrees)	average value of the thickness h_L (cm)	F: frequency of the thickness oscillations (Hz)
31	8.5	0.50
31	7.3	0.39
33	7.4	0.44
33	8.35	0.48
35	8.25	0.55
35	7.6	0.50
37	8	0.68
37	9.55	0.84
39	6.25	0.42
39	6.05	0.38
39	6.8	0.50
39	9.35	0.68

Table 1: Slope angle, average thickness and frequency of the thickness oscillations for each analyzed flow.

a Fourier transform of the signal received from the thickness's sensor.

2.2 Thickness oscillations, velocity profile and mean velocity

Initially the channel is empty. The beginning of the experiment consists of a transient phase, while the front of the flow goes throughout the channel. After this transient phase, as shown in figure 2, at any given location the flow thickness oscillates in time around a mean value $\langle h_x \rangle$ defined as a time average (steady component of the flow). As also illustrated in figure 2, the variations of h_L follows those of $h_{L-1.5}$ with a constant delay τ : $h_L(t) \approx h_{L-1.5}(t - \tau)$. It thus appears that the steady component of the flow is x-uniform at the location of the probes. The frequency of the thickness oscillations mentioned above is always close to the rotation speed of the screw, as illustrated by figure 3. Indeed, the rotation speed of the screw can be calculated

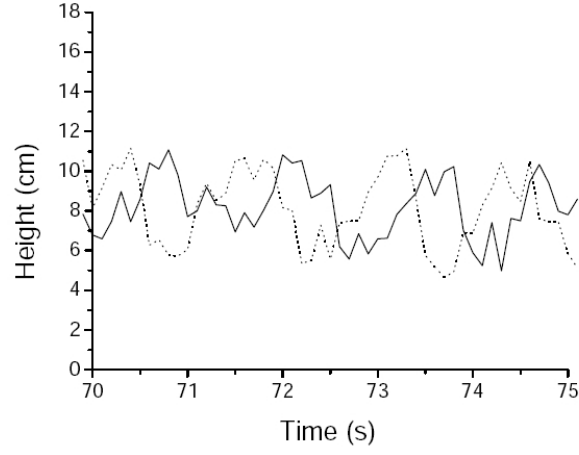


Figure 2: Example of thickness variations. h_L (full line) is measured 1,5 m downstream from $h_{L-1.5}$ (dashed line). The figure corresponds to a laps of 10 seconds taken from a flow lasting approximately 30 seconds.

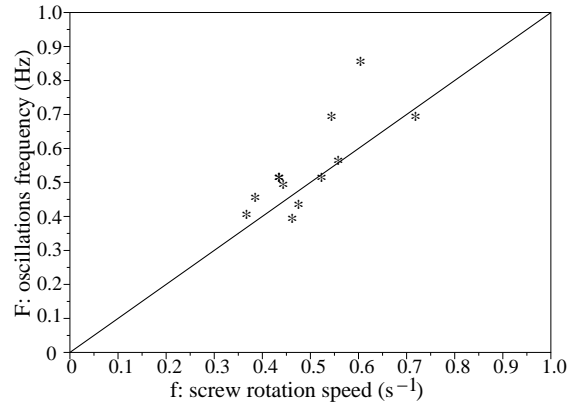


Figure 3: Frequency (F) of the oscillations observed as a function of the screw rotation speed (f). The straight line is given by $F = f$.

according to the formula: $f = \mathcal{V}^{-1} \times Q$ with Q the flow rate (obtained from the velocity profile as described below) and $\mathcal{V} \approx 0.14 \text{ m}^3$ (information given by the manufacturer). f is calculated with a 10% error.

Special attention was payed to side wall effects on the flow. First the existence of a transverse velocity gradient was qualitatively looked for. To that purpose the flow was filmed from above. The observation show a displacement of the snow as a block without slowdown toward the walls. More quantitatively Louge and Keast (2001) measured velocity gradients in a granular channelled flow and obtained less than 10% in velocity between the center of the flow and the side walls. Because in the present case the material is cohesive the gradient should be even less than these 10%. Side walls could also be responsible for a global slowdown of the flow. Ericksson (1955) performed measurments of friction coefficients between snow and different materials. The friction coefficient between snow and plexiglas was measured to be around 0.05 which is ten times lower than the friction between snow and the sandpaper stuck on the bottom of the channel. Thus, it can be assumed that the effect on the mean velocity is negligible. From these two points, the flow can be considered as uniform in the direction perpendicular to the flow.

The mean velocity profile is written $V(z)$, where z is the distance from the bottom of the channel. The results for the twelve different flows are all satisfyingly fitted by a linear profile (Bouchet et al., 2004):

$$V(z; \theta, h) = V_s + \Gamma_0 z \quad (1)$$

where:

$$V_s(\theta) = V_0 [\tan \theta - \tan \theta_0] \quad (2)$$

and

$$\begin{cases} V_0 &= 9.1 \text{ m.s}^{-1} \\ \tan \theta_0 &= 0.28 \ (\theta_0 = 15.6^\circ) \\ \Gamma_0 &= 12.5 \text{ s}^{-1} \end{cases} \quad (3)$$

Such a linear profile is not usual but has already been observed in some granular flows (Douady et al., 2002; Bonamy et al., 2002a). Bonamy et al. (2002b) explained this uniformity of the velocity gradient by the presence, in the flow, of aggregates of many different sizes without one typical scale. In the present flow, aggregates were also detected (Bouchet, 2003a). It should be mentioned that the behavior of the flow near the ground was not explored. Indeed, the first transducer is located more than ten to twenty particles diameters above the ground. Thus, it is here impossible to know if there is a basal sliding or a strong basal shear. The presence of the sandpaper on the bottom makes the second hypothesis the more believable. Nevertheless in the present article we are only interested in the averaged velocity over the flow thickness which is quite insensitive to this basal behavior. Moreover, during the numerical calculation the shape factor is taken equal to one (see section 3).

The integral of equation (1) gives the averaged velocity over the flow thickness:

$$u(\theta, h) = V_s(\theta) + \frac{1}{2} \Gamma_0 h \quad (4)$$

or, using equation 2,

$$u = V_0 [\tan \theta - \tan \theta_0] + \frac{1}{2} \Gamma_0 h. \quad (5)$$

2.3 Friction law

Because the mean flow near the probes is fully developed and x-uniform, the friction force exerted by the channel on the flow balances the weight of the snow. The effective friction coefficient μ , defined as the ratio between the shear stress and the normal stress at the bottom of the channel,

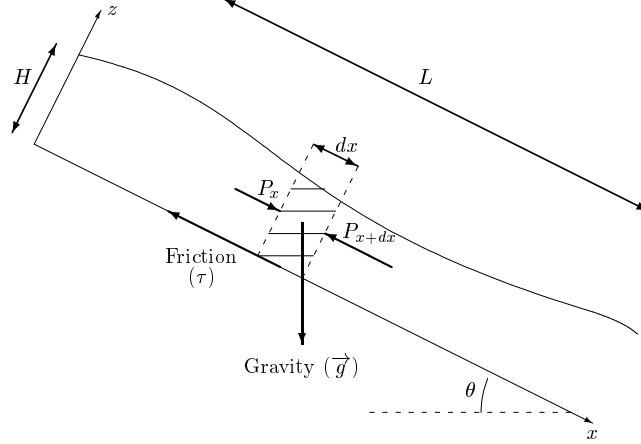


Figure 4: Sketch of the flow and of the applied forces.

is thus equal to the tangent of the slope angle θ . Using equation (5), the friction law can be written as:

$$\mu = \mu_0 + \frac{u}{V_0} - \frac{h}{H_0} \quad (6)$$

with

$$\begin{cases} \mu_0 &= \tan \theta_0 &= 0.28 \\ H_0 &= 2V_0/\Gamma_0 &= 1.45 \text{ m} \end{cases} \quad (7)$$

It must borne in mind that this friction law is here established for the particular dynamical conditions of a fully developed and x-uniform flow. Another point is that this friction law has been established in a given range of slope angles and flow thicknesses corresponding to those of the experiments and should not be extrapolated to some others too far as for example too large values of the thickness. The non-transient phase of the following simulations is performed close to this range of validity. The initial transient phase corresponds to small flow thicknesses. Thus, the friction law is supposed valid even during this phase in our simulations.

3 Study of the steady component of the flow

In this section we study in detail the behavior of the steady part of the fully developed flow. To that purpose we use the laterally unconfined Saint-Venant equations solving them first numerically and in a second time analytically.

3.1 The Saint-Venant equations

Originally used for hydraulic studies of shallow water flows, the Saint-Venant equations were first applied to granular flows by Savage and Hutter (1989). This approach is now commonly employed to model dense snow flows. The equations are derived by averaging the mass and momentum conservation equations over the flow thickness. Two assumptions are employed:

- the typical flow thickness is assumed to be small compared to the streamwise variations (short wavelength hypothesis);
- the flow density is taken as constant and uniform.

Hence, the dominant component of flow velocity, u , is parallel to the bottom of the channel. The flow is described by its thickness $h(x, t)$ and average velocity $u(x, t) = (\int_0^h u(x, z, t) dz)/h$. Mass and momentum equations for a thin slice can be written:

$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} = 0 \quad (8)$$

$$\frac{\partial hu}{\partial t} + \alpha \frac{\partial hu^2}{\partial x} = gh \cos \theta (\tan \theta - \mu - k \frac{\partial h}{\partial x}) \quad (9)$$

where

- α is the shape factor: $\alpha = h \int_0^h u^2 dz / \left[\int_0^h u dz \right]^2$;
- k is the ratio between the normal stress parallel to the ground, σ_{xx} , and the normal stress perpendicular to the ground, σ_{zz} .

In what follows, the stress tensor is supposed to be isotropic and thus k is taken equal to one. In granular flows, the precise value of k does not appreciably affect the flow behavior. For commonly encountered velocity profiles (uniform, linear, parabolic), α takes values between 1 and 2. The flow is also insensitive to the precise value used for α ; here we take $\alpha = 1$. Finally friction law (6) is used.

3.2 Description of the numerical solution procedure and result

The 6-meter long channel upstream of the measuring device is discretized in x with points every five centimeters. The time step is $dt = 5 \times 10^{-3}$ s, and we run the model for thirty seconds. The numerical scheme used is an explicit finite difference MacCormack scheme.

Both steady-inlet and initial conditions are needed for h and u . As regards the steady-inlet conditions ($h(0)$, $u(0)$), once the flow becomes steady equation (8) implies a constant volume flux:

$$h(0)u(0) = h(L, t)u(L, t) = Q \quad (10)$$

Experimental values of h_L and u_L (given by (5)) allow Q to be determined. $h(0)$ is taken as an input parameter for the simulation and $u(0)$ calculated using $u(0) = Q/h(0)$. These constant values of $h(0)$ and $u(0)$, which apply strictly to the steady component of the flow, are imposed throughout the calculation. As can be observed in section (3.3) the value used for $h(0)$ has no influence on the asymptotic flow of the end of the channel.

The values used for θ and h_L are the mean of those encountered during the experiments:

$$h_L = 7.6 \text{ cm} \quad (11)$$

and

$$\theta = 35^\circ \quad (12)$$

When the model is run, an initial unsteady, non-uniform phase is observed, corresponding to the passage of the front along the channel (see figure 5). This phase is not discussed because the Saint-Venant equations cannot deal well with rapid changes in thickness and velocity. The only thing that can be noticed is that the passage of the transient phase lasts around 2 to 3 seconds both in the experiments and in the numerics. Once the front has passed, a fully developed steady regime is observed (see for example figure 6). Near the end of the channel the flow is found to be x-uniform. This is an important point for the exploitation of the experimental results in general and of the friction law in particular. This regime is examined in more detail in the following section. Just for information, the initial values are $h(x > 0, t = 0) = 1$ cm and $u(x > 0, t) = 0$. The non-zero value of h was introduced to avoid a division by zero. We choose these values for the sake of simplicity and because the fully developed flow (the one which is relevant to study here) is not sensitive to them.

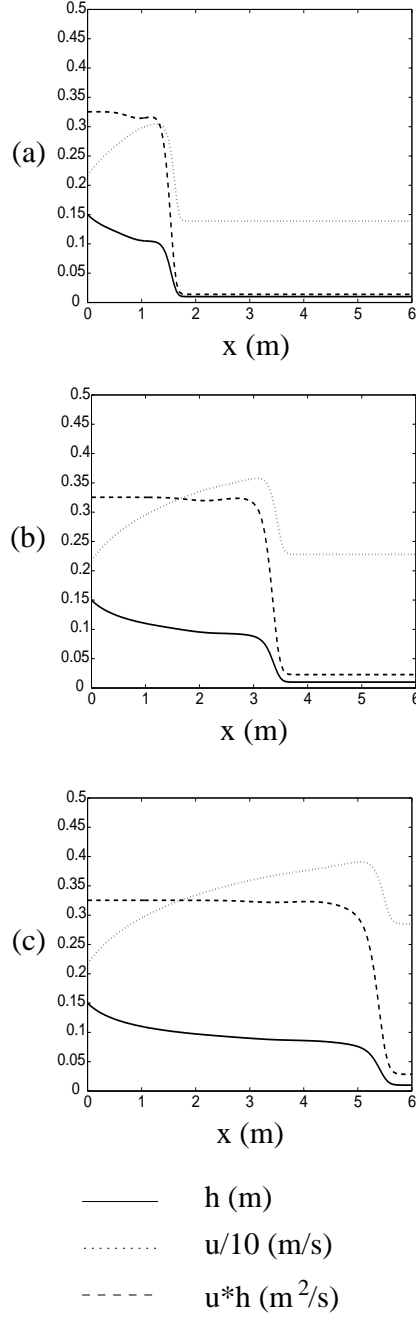


Figure 5: Saint-Venant model's results using $h_0 = 15$ cm. thickness h (in m), velocity $u/10$ (in m/s) and flow rate $u \cdot h$ (in m^2/s) are plotted versus position x . These results concern the beginning of the flow when the front is crossing the channel before the establishment of the fully developed regime. (a): $t = 0.5$ s, (b): $t = 1$ s, (c): $t = 1.5$ s.

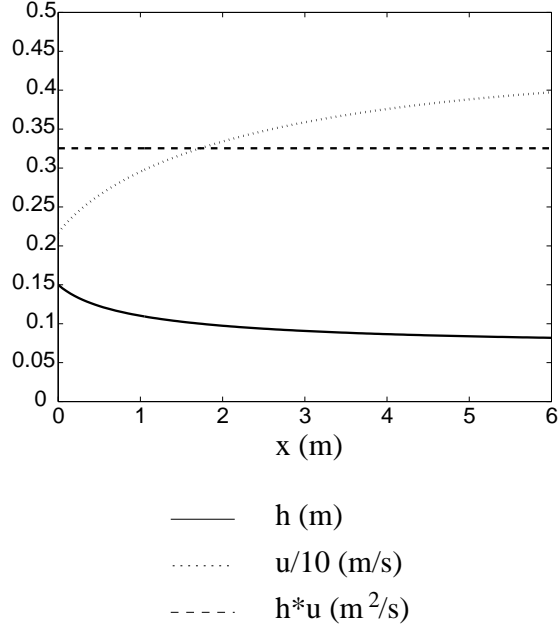


Figure 6: Saint-Venant model's results using $h_0 = 15$ cm, thickness h (in m), velocity $u/10$ (in m/s) and flow rate uh (in m^2/s) are plotted versus position $x(\text{m})$. These results are obtained in the fully developed regime.

3.3 Analytical study of the steady component of the flow

In this section, we solve analytically the Saint-Venant equation for the steady component of the flow. This enables us to quantify when the asymptotic x-uniform regime is reached. For steady flows, the Saint-Venant equations (equations (8) and (9)) take the form:

$$uh = Q \quad (13)$$

$$\alpha \frac{dh u^2}{dx} = gh \cos \theta (\tan \theta - \mu_0 - \frac{u}{V_0} + \frac{h}{H_0} - k \frac{dh}{dx}) \quad (14)$$

3.3.1 Values of the x-uniform asymptotic regime

Let h_∞ and u_∞ be the flow thickness and flow velocity when the x-uniform, asymptotic regime is reached. From equations (13) and (14), h_∞ and u_∞ are given by:

$$\begin{cases} h_\infty u_\infty = Q \\ \mu_0 + \frac{u_\infty}{V_0} - \frac{h_\infty}{H_0} = \tan \theta \end{cases} \quad (15)$$

This system admits only one set of positive solutions:

$$\begin{cases} h_\infty &= \frac{H_0(\tan \theta - \mu_0)}{2} (-1 + \sqrt{1 + \varepsilon}) \\ u_\infty &= \frac{V_0(\tan \theta - \mu_0)}{2} (1 + \sqrt{1 + \varepsilon}) \end{cases} \quad (16)$$

where

$$\varepsilon = \frac{4Q}{V_0 H_0 (\tan \theta - \mu_0)^2} \quad (17)$$

3.3.2 Distance needed to reach the x-uniform asymptotic regime

We next study the development of the flow with streamwise distance x . To that purpose we integrate the differential equations (13) and (14) analytically using the following reduced variables: $\tilde{h} = (h - h_\infty)/h_\infty$, $\tilde{u} = (u - u_\infty)/u_\infty$, with $\tilde{x} = (x + x_0)/h_\infty$ with x_0 a constant value. Details can be found in appendix 1. The integration leads to

$$\begin{aligned} \ln \left[\left| \tilde{h} \right|^{k - \alpha F r^2} \left| \tilde{h} + 1 \right|^{\alpha F r^2 (1 - 1/\beta^2)} \left| \tilde{h} + 1 + \beta \right|^{k\beta + \alpha F r^2 / \beta^2} \right] \\ - \alpha F r^2 \frac{1 + \beta}{\beta} \frac{1}{\tilde{h} + 1} = (1 + \beta) \frac{h_\infty}{H_0} \tilde{x} \end{aligned} \quad (18)$$

with

$$\begin{cases} h_\infty &= \frac{H_0 (\tan \theta - \mu_0)}{2} [-1 + \sqrt{1 + \varepsilon}] \\ u_\infty &= \frac{V_0 (\tan \theta - \mu_0)}{2} [1 + \sqrt{1 + \varepsilon}] \\ \varepsilon &= \frac{4Q}{V_0 H_0 (\tan \theta - \mu_0)^2} \\ F r^2 &= \frac{u_\infty^2}{g h_\infty \cos \theta} \\ \beta &= \frac{u_\infty H_0}{h_\infty V_0} = \frac{(1 + \sqrt{1 + \varepsilon})^2}{\varepsilon} \end{cases} \quad (19)$$

and

$$x_0 = \frac{H_0}{1 + \beta} \times (\ln[|\tilde{h}(0)|^{k - \alpha F r^2} |\tilde{h}(0) + 1|^{\alpha F r^2 (1 - 1/\beta^2)} |\tilde{h}(0) + 1 + \beta|^{k\beta + \alpha F r^2 / \beta^2}] - \frac{\alpha F r^2 (1 + \beta)}{\beta(\tilde{h}(0) + 1)}) \quad (20)$$

Using equation (18) we plot \tilde{h} as a function of \tilde{x} in figure 7. The entire curve has three branches. Because the thickness of the flow is limited by the vertical size of the channel ($h < 20$ cm) and because the flow thickness decreases experimentally throughout the flow we only keep the part of the curve which fits these conditions. Looking at the curve, one can see that when \tilde{x} becomes large enough, the flow reaches asymptotically its x-uniform state. This confirms what the numerics seem to predict. The question is now to know whether the asymptotic x-uniform regime is reached before the end of the channel. To that purpose we simplify equation (18) for the case of large \tilde{x} . It can be rewritten in:

$$\tilde{h} = (1 + \beta)^{-(k\beta + \frac{\alpha F r^2}{\beta})} \exp \left(-\frac{(1 + \beta) h_\infty}{H_0 (\alpha F r^2 - k)} \tilde{x} \right) \quad (21)$$

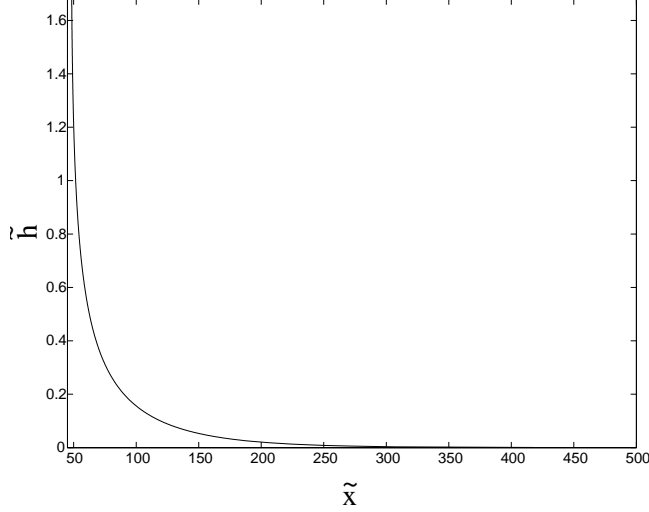


Figure 7: Plot of the relative deviation of the flow thickness \tilde{h} as a function of the relative distance \tilde{x} .

The typical length scale is then:

$$L = \frac{H_0(\alpha Fr^2 - k)}{(1 + \beta)} = 4 \text{ m} \quad (22)$$

when taking the experimental value $Q = 0.32 \text{ m}^2.\text{s}^{-1}$. It is therefore relevant to consider the flow as x-uniform at the location of the measuring device, 6 m downstream from the top of the channel.

4 Origin of the oscillations - validation of the model

In this section we show that the model gives an explanation about the origin of the oscillations observed in the experiments.

4.1 Frequency of the oscillations

The oscillations induced by the screw which feeds snow into the channel are modeled by the boundary condition $h(0, t) = h_0 + h_1 \cos(\omega t)$ with $f = \omega/2\pi$ the frequency of the screw. To study what happens at the end of the channel we use the following scheme:

- First, the model is run for 5 seconds with $h(0, t) = h_0 + h_1 \cos \omega t$ to allow time for the initial transient phase to have finished.
- After these 5 seconds the model is run for a further 30 seconds and the temporal Fourier transform of $h(L, t)$ is taken.

An example of the results is shown in figure 8, which also gives the result for $h(0, t)$. In addition to the fundamental frequency f and the zero frequency peak representing the mean value, the harmonics $2f, 3f, \dots$ are also present in $h(L, t)$ due to nonlinearity of equations (8) and (9). The amplitude of these frequency components is always less than 10% of the amplitude of the fundamental peak.

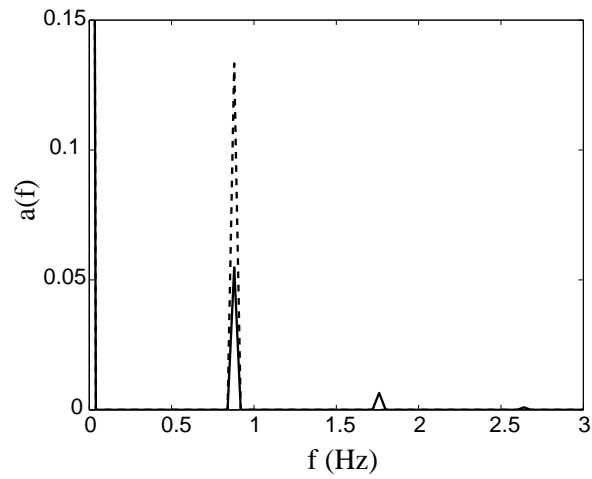


Figure 8: Fourier-transform amplitude of the thickness of the flow obtained by the Saint-Venant model. --- Fourier-transform amplitude of $h(0, t)$, — Fourier-transform amplitude of $h(L, t)$.

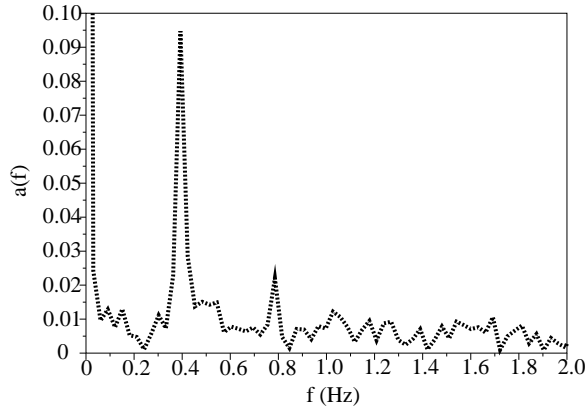


Figure 9: Fourier-transform amplitude of $h(L, t)$ with $h(L, t)$ measured during one experiment.

4.2 Comparison between model and experiments

The Fourier transform of $h(L, t)$ in the experiments also shows the presence of the frequencies $2f, 3f \dots$ (cf figure (9)). However, the ratio of the amplitude of these secondary peaks to that of the fundamental differs in the model and in the experiments. In both cases (numerical and experimental) the secondary peaks are weak and can thus be neglected.

Let us now compare the ratio of the amplitude of the fundamental peak to that at zero frequency. This ratio is plotted in figure 10. With $h_0 = 15$ cm and $h_1 = 4$ cm the model gives good results compared with the experimental values. The curves given by the extreme values of the parameters are also plotted on figure 10 ($h_0 = 12$ cm, $h_1 = 6$ cm (eg) $h_0/h_1 = 2$ and $h_0 = 16$ cm, $h_1 = 2$ cm (eg) $h_0/h_1 = 8$). By “extreme values” we mean the larger or smaller values of the parameters the experimental observations allow to use. When plotting the curves what appears to be important is not the values of h_0 and h_1 separately but the ratio h_0/h_1 . Indeed, with the same ratio, two sets of values $(h_{0a}, h_{1a}), (h_{0b}, h_{1b})$ give merely the same curves. This is the reason why we have plotted only two “extreme” cases. One can see that if the experimental values are close to the predictive curve they are also all included in the space between the two extreme curves. This allows for different conclusions. The majority of points (those that are close to the predictive curve) show that the model well predicts the behavior of

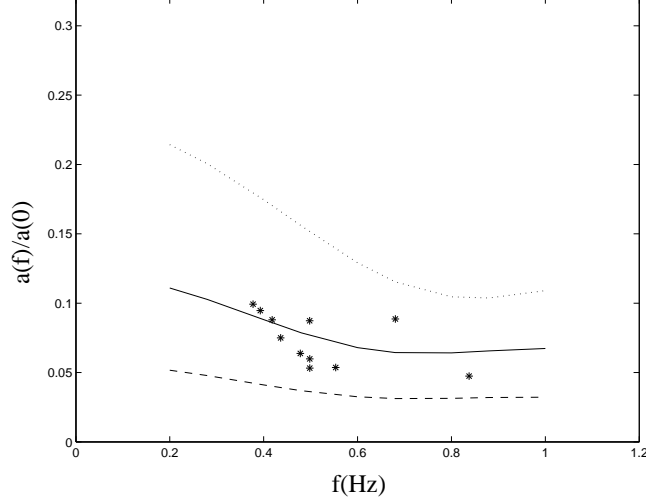


Figure 10: Ratio between the amplitude of the fundamental peak and the amplitude of the peak at zero frequency versus frequency. * results from experiments, — results of the Saint-Venant model with $h_0 = 15$ cm and $h_1 = 4$ cm, results of the Saint-Venant model with $h_0 = 12$ cm and $h_1 = 6$ cm, - - - results of the Saint-Venant model with $h_0 = 16$ cm and $h_1 = 2$ cm.

the flow. The model is run in each case with fixed parameters ($\theta = 35^\circ$, $h_L = 7.6$ cm). These values are the mean of the 11 first experimental values (see table 1), the twelfth amplitude was not usable. Hence it seems reasonable that some of the points are not strictly close to the predictive curve. Nevertheless they are still between the extreme curves. All of this confirm the hypothesis that the model well predicts the phenomenon and the flow and that the oscillations are due to the screw.

5 Conclusion

The dense snow flows studied during the experiments exhibit thickness's oscillations. Though experimental facts suggest that they are generated by the feeding system of the set-up, the origin of these oscillations remain *a priori* uncertain. The friction law for dense snow flows deduced from the experiments is used in numerical resolutions of Saint-Venant's equations. The initial conditions given to the thickness at the entrance of the channel consist in sinusoidal oscillations with an amplitude h_1 , around a mean value h_0 . The simulations performed lead to the following results:

- the mean flow is x-uniform at the location of the measuring device. This is a crucial point for the validation of the results of the experiments in which the hypothesis of x-uniformity was the central hypothesis;
- the oscillations engendered at the channel's entrance propagate and are still present with the same frequency (harmonics can be neglected) at the measurements' position;
- numerical and experimental results are in a good agreement when using an oscillation ratio $h_1/h_0 = 4/15$. From direct observations, this ratio seems very reasonable;
- the relative intensity of these oscillations (given by the FFT of $h(x, t)$) reduces from about 15 % at the top of the channel to approximatively 5 % to 10 % at the studied position.

The results presented in this paper agree with the experimental observations. They suggest that:

1. in the experiments, the mean value of the thickness of the flow at the entrance of the channel can be changed as wanted without affecting the x-uniformity of the flow at the end of the channel and thus the validity of the measurements;
2. reasonable oscillations generated by the feeding system can propagate all along the channel and have the same intensity as the one observed in the experimental results;

The present study reveals that, as many other flows, such channeled flows of dry snow can be described using a Saint-Venant like model. Such numerical studies can help in the future in preparing other configurations of the device to test other characteristics of those channeled snow flows and thus rheology of snow.

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Appendix 1

Using the mass conservation equation (equation (13)), equation (14) can be rewritten in:

$$\frac{d}{dx} \left[\alpha Q u + \frac{kg \cos \theta}{2} h^2 \right] = gh \cos \theta \left[\tan \theta - \mu_0 - \frac{u}{V_0} + \frac{h}{H_0} \right] \quad (23)$$

Let δu and δh be the deviations from the x-uniform regime:

$$\begin{cases} u &= u_\infty + \delta u \\ h &= h_\infty + \delta h \end{cases} \quad (24)$$

Equation (23) becomes:

$$\frac{d}{dx} \left[\alpha Q \delta u + \frac{kg \cos \theta}{2} (2h_\infty \delta h + \delta h^2) \right] = g(h_\infty + \delta h) \cos \theta \left[\frac{\delta h}{H_0} - \frac{\delta u}{V_0} \right] \quad (25)$$

We now turn all lengths and velocities without dimension by dividing them respectively by h_∞ and by u_∞ and define:

$$\begin{cases} \tilde{x} &= x/h_\infty \\ \tilde{h} &= (h - h_\infty)/h_\infty \\ \tilde{u} &= (u - u_\infty)/u_\infty \end{cases} \quad (26)$$

Mass conservation (equation (13)) implies:

$$\tilde{h}\tilde{u} + \tilde{h} + \tilde{u} = 0 \quad (27)$$

At this stage let's notice that if \tilde{h} tends to zero, because of equation (27), \tilde{u} also tends to zero (consequence of mass conservation) (eg) the x-uniform regime is reached. Hence in what follows we focus our study on \tilde{h} . For its part, equation (25) becomes:

$$\frac{d}{d\tilde{x}} \left[\alpha Fr^2 \tilde{u} + k\tilde{h} + \frac{1}{2}k\tilde{h}^2 \right] = (1 + \tilde{h}) \left(\frac{h_\infty}{H_0} \tilde{h} - \frac{u_\infty}{V_0} \tilde{u} \right) \quad (28)$$

with Fr the Froude number of the x-uniform regime:

$$Fr = \frac{u_\infty}{\sqrt{gh_\infty \cos \theta}} \quad (29)$$

Substituting, in equation (28), \tilde{u} by its expression obtained from equation (27) and introducing the parameter $\beta = u_\infty H_0 / (h_\infty V_0)$, gives:

$$\frac{d}{d\tilde{x}} \left[k\tilde{h} + \frac{k}{2}\tilde{h}^2 - \alpha Fr^2 \frac{\tilde{h}}{\tilde{h} + 1} \right] = \frac{h_\infty}{H_0} \tilde{h} (\tilde{h} + 1 + \beta) \quad (30)$$

which can be rewritten in:

$$\left[k \frac{\tilde{h} + 1}{\tilde{h}(\tilde{h} + 1 + \beta)} - \alpha Fr^2 \frac{1}{\tilde{h}(\tilde{h} + 1 + \beta)(\tilde{h} + 1)^2} \right] d\tilde{h} = \frac{h_\infty}{H_0} d\tilde{x} \quad (31)$$

A straightforward integration of equation (31) leads to:

$$\ln \left[\left| \tilde{h} \right|^{k-\alpha Fr^2} \left| \tilde{h} + 1 \right|^{\alpha Fr^2(1-1/\beta^2)} \left| \tilde{h} + 1 + \beta \right|^{k\beta + \alpha Fr^2/\beta^2} \right] - \alpha Fr^2 \frac{1+\beta}{\beta} \frac{1}{\tilde{h}+1} = (1+\beta) \frac{h_\infty}{H_0} \tilde{x} + cst \quad (32)$$

with

$$\left\{ \begin{array}{lcl} h_\infty & = & \frac{H_0 (\tan \theta - \mu_0)}{2} [-1 + \sqrt{1 + \varepsilon}] \\ u_\infty & = & \frac{V_0 (\tan \theta - \mu_0)}{2} [1 + \sqrt{1 + \varepsilon}] \\ \varepsilon & = & \frac{4Q}{V_0 H_0 (\tan \theta - \mu_0)^2} \\ Fr^2 & = & \frac{u_\infty^2}{gh_\infty \cos \theta} \\ \beta & = & \frac{u_\infty H_0}{h_\infty V_0} = \frac{(1 + \sqrt{1 + \varepsilon})^2}{\varepsilon} \end{array} \right. \quad (33)$$

References

- D. Bonamy, F. Daviaud, and L. Laurent. Experimental study of granular surface flows via a fast camera: a continuous description. *Phys. Fluids*, 14:1666–1673, 2002a.
- D. Bonamy, F. Daviaud, L. Laurent, M. Bonetti, and J.P. Bouchaud. Multi-scale clustering in granular surface flows. *Phys. Rev. Letters*, 89:034301, 2002b.
- A. Bouchet. *Etude expérimentale des avalanches denses de neige sèche*. PhD thesis, UJF–Grenoble, 2003a.
- A. Bouchet, M. Naaim, H. Bellot, and F. Ousset. Experimental study of dense snow avalanches: velocity profiles in quasi-permanent and fully developed flows. *Ann. Glaciology*, 38:30–34, 2004.
- A. Bouchet, M. Naaim, F. Ousset, H. Bellot, and D. Cauvard. An experimental set-up to study the flow of dense and dry snow: presentation of the set-up and first results. *Surv. Geophys.*, 24:525–541, 2003b.
- G. Casassa, H. Narita, and N. Maeno. Shear cell experiments of snow and ice friction. *J. Appl. Phys.*, 69 (6):3745–3755, 1991.
- J. D. Dent, K. J. Burrell, D. S. Schmidt, M. Y. Louge, E. E. Adams, and T. G. Jazbutis. Density, velocity and friction measurements in a dry-snow avalanche. *Ann. Glaciology*, 26:247–252, 1998.
- J. D. Dent and T. E. Lang. A biviscous modified bingham model of snow avalanche motion. *Ann. Glaciology*, 4:42–46, 1983.
- S. Douady, B. Andreotti, A. Daerr, and P. Clad. From a grain to avalanches: on the physics of granular surface flows. *C. R. Physique*, 3:1–10, 2002.
- R. Ericksson. Medens friktion mot sn och is (friction of runners of snow and ice). *U. S. army snow ice and permafrost research establishment*, Translation44, 1955.
- M. A. Kern, F. Tiefenbacher, and J. N. McElwaine. The rheology of snow in large chute flows. *Cold Region Sci. Tech.*, 39:181 – 192, 2004.
- M. Y. Louge and S. C. Keast. On dense granular flows down flat frictionnal inclines. *Phys. Fluids*, 13-5:1213–1233, 2001.
- M. Naaim, T. Faug, and F. Naaim-Bouvet. Dry granular flows: erosion and deposition modelling. *Surv. Geophys.*, 24:569–585, 2003.
- K. Nishimura and N. Maeno. Contribution of viscous forces to avalanche dynamics. *Ann. Glaciology*, 13:202–206, 1989.
- H. Norem, F. Irgens, and B. Schieldrop. A continuum model for calculating snow avalanche velocities. *Int. Assoc. Hydrol. Sci. Pub.*, 162:363–379, 1987.
- R. Perla, T. T. Cheng, and D. M. McClung. A two-parameter model of snow-avalanche motion. *J. Glaciology*, 26:197–207, 1980.
- S. B. Savage and K. Hutter. The motion of a finite mass of granular material down a rough incline. *J. Fluid Mech.*, 199:177–215, 1989.
- A. Voellmy. ber die zerstrungskraft von lawinen. *Schweiz. Bauzeitung*, 73:159–165, 212–217, 246–249, 280–285, 1955.